**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-1, Algebra**

**Assignment-1**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following:** **(10 Marks)**

1. (a) What do you mean by semigroup? State and prove generalized associative law in a semigroup.

   (b) State and prove generalized commutative law in a commutative semigroup.

2. Define the concept of an ideal of a ring. State and prove Chinese Remainder theorem.

## Section: B

**Answer any one of the following:** **(5 Marks)**

3. Let H, K be normal subgroups of a group G such that $K \subseteq H$. Show that H/K is a normal subgroup of G/K and $(G/K) / (H/K) \cong G/H$.

4. State and prove first sylow's theorem.

**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-1, Algebra**

**Assignment-2**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following: (10 Marks)**

1. Prove that every principal ideal domain is a unique factorization domain.
2. State and prove fundamental theorem of Galois Theory.

## Section: B

**Answer any one of the following: (5 Marks)**

3. If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)\,g(x)$ is also a primitive polynomial.
4. If $E$ is a finite normal extension of a field $F$, then prove that $E$ is a splitting field of some polynomial $f(x) \in F[X]$.

**Dr.B.R.Ambedkar Open University**

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I year M.Sc (Mathematics) 2022-23

**Course-2, Real Analysis**

**Assignment-1**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following:** **(10 Marks)**

1. (a) If $f$ is a mapping from a metric space $X$into a metric space $Y$ and $x \in X$, then show that $f$ is continuous at $x$ if and only if $\{f(x_n)\}$ converges to $f(x)$ for any sequence $\{x_n\}$ converges to $x$.

   (b) If $f$ is a function from a metric space $X$ into a metric space $Y$ , then show that $f$ is Continuous on $X$ if and only if $f^{-1}(G)$ is open in $X$ whenever $G$ is open in $Y$.

2. State and prove necessary and sufficient condition for a bounded function $f$ on $[a, b]$ to be Riemann-stieltjes integrable.

## Section: B

**Answer any one of the following:** **(5 Marks)**

3. If an infinite product $\prod_{n=1}^{\infty} a_n$ is convergent, then show that $a_n \to \infty\ as\ n \to \infty$.

4. Show that, for all $x > 0, y > 0$, $B(x, y) = 2 \int_0^{\frac{\pi}{2}} sin^{2x-1}\theta\ \ cos^{2y-1}\theta\ d\theta.$

**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-2, Real Analysis**

**Assignment-2**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following: (10 Marks)**

1. If $f$ is a continuous real function on [0,1] and $\delta > 0$, then show that there is a polynomial $B$ such that $|f(x) - B(x)| < \delta$ for all $x \epsilon [0,1]$.

2. (a) If $f$ and $g$ are measurable functions then prove that $f+g$ and $fg$ are measurable.

   (b) Give an examples of a function $f$ for which $|f|$ is measurable, but $f$ is not.

## Section: B

**Answer any one of the following: (5 Marks)**

3. Define the Gamma function and prove that $\Gamma(x+1) = x\,\Gamma(x)$ for all $x > 0$.

4. If $f$ is bounded and $R$ – integrable on $[a, b]$ then show that $f$ is $L$- integrable and that the $L$- integral and $R$- integral of $f$ over $[a, b]$ are equal.

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I year M.Sc (Mathematics) 2022-23

**Course-3, Discrete Mathematical Structures**

**Assignment-1**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following:** **(10 Marks)**

1. (a) Define the notion of a lattice homomorphism and give two examples.

   (b) Let $L$ and $M$ be lattices and $f: L \to M$ a bijection. Prove that $f$ is a lattice isomorphism if and only if both $f$ and $f^{-1}$ are isotones.

2. (a) Construct the truth table for the following statement formulae.

   (i) $(P \to Q) \wedge ((\urcorner P) \to R))$ (ii) $\urcorner(P \vee (Q \wedge R) \leftrightarrows (P \vee Q) \wedge (P \vee R)$

   (b) Define the tautology. Determine whether the following statement formulae are tautologies or contradictions.

   (i) $P \vee (\urcorner P)$ (ii) $\urcorner(P \vee Q) \vee ((\urcorner P) \wedge Q) \vee P$

## Section: B

**Answer any one of the following:** **(5 Marks)**

3. Define an atom. Prove that a non zero element $p$ in a Boolean algebra $B$ is an atom if and only if, for any $a \in B$ either $p \wedge a = 0$ or $p \leq a$.

4. Find the disjunctive normal form of the formula $P \wedge (P \to Q)$.

**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-3, Discrete Mathematical Structures**

**Assignment-2**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following:** **(10 Marks)**

1. (a) State and prove Euler's theorem on degrees of vertices in a graph.

   (b) State and prove a necessary and sufficient condition for a graph to be a connected graph.

2. (a) Design a TM which accepts all strings of the form $a^n b^n$ for $n \geq 1$.

   (b) State and prove pumping lemma for regular languages.

## Section: B

**Answer any one of the following:** **(5 Marks)**

3. (a) Show that every connected graph has a spanning tree.

   (b) Discuss Konigsberg Bridge problem.

4. (a) Define a string and the length of a string with the usual notation . Let *u* and *v* be any two Strings of $\sum^*$. Then show that $|uv| = |u| + |v|$.

   (b) Solve the recurrence relation $a_n = a_{n-1} - 2\ ;\ a_1 = 0$ by iterative method.

Dr.B.R.Ambedkar Open University
Faculty of Science, Department of Mathematics
I year M.Sc (Mathematics) 2022-23

**Course-4, Differential Equations**

**Assignment-1**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following: (10 Marks)**

1. (a) Let $\Phi_1, \Phi_2, \ldots\ldots, \Phi_n$ be solutions of $L(y) = 0$ on $I$, an interval of $R$ and $x_0 \in I$, then show that $W(\Phi_1, \Phi_2, \ldots\ldots, \Phi_n)(x) = W(\Phi_1, \Phi_2, \ldots\ldots, \Phi_n)(x_0)\, e^{-a_1(x-x_0)}$ $\forall\, x \in I$.

   (b) Solve the initial value problem

   $$y^{(4)} + y = 0, y(0) = 1, y'(0) = 0, y''(0) = -1, y'''(0) = 1$$

2. Prove that for any non- negative integer $n$,

$$p_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \quad \text{for all} \quad x \in (-\infty, \infty)$$

## Section: B

**Answer any one of the following: (5 Marks)**

3. Find all solutions of the differential equation

   $y''' + y'' + y' + y = x^2 e^x$ on $R$ by using the method of variation of parameters.

4 . Prove orthogonality property of chebyshev polynomials.

**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-4, Differential Equations**

**Assignment-2**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following: (10 Marks)**

1. State and prove Liouville's Theorem.
2. Determine the type of stability of the critical point (0, 0) of the system

$$\frac{dx}{dt} = -y - x^2 \quad , \quad \frac{dy}{dt} = x$$

## Section: B

**Answer any one of the following: (5 Marks)**

3. (a) Form a PDE by eliminating arbitrary constants : $z = ax^3 + by^3$

   (b) Form a PDE by eliminating arbitrary function: $\phi(x^2 + y^2 - 2z, xyz) = 0$

4. Solve (a) $(y + zx)p - (x + yz)q + y^2 - x^2 = 0$

   (b) $p^2x + q^2 y = z$ by using Charpit's method.

**Dr.B.R.Ambedkar Open University**
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I year M.Sc (Mathematics) 2022-23
**Course-5, Topology**
**Assignment-1**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following: (10 Marks)**

1. (a) Show that every separable metric space is second countable.

   (b) Let $f$ be a mapping of a metric space $X$ into another metric space $Y$. Show that $f$ is continuous at a point $x_0$ in $X$ if and only if for every sequence $\{x_n\}$ in X with $x_n \to x_0$. we have $f(x_n) \to f(x_0)$.

2. Prove that a metric space is sequentially compact ⇔ it satisfies the Bolzano-Weierstrass property.

## Section: B

**Answer any one of the following: (5 Marks)**

3. Suppose $X$ is a topological space $A \subseteq X$. Then prove the following
   a) $A$ is open ⇔ Int ($A$) = $A$.
   b) $A$ is closed ⇔ b(A).
   c) $x \in b(A)$ ⇔ each neighbourhood of $x$ intersects both A and $A^c$.

4. State and prove Lindelif 's theorem.

**Dr.B.R.Ambedkar Open University**

Faculty of Science, Department of Mathematics

I year M.Sc (Mathematics) 2022-23

**Course-5, Topology**

**Assignment-2**

Max.Marks:15 Min.Marks:6

## Section: A

**Answer any one of the following:** **(10 Marks)**

1. State and prove Tietze's extention theorem.

2. Prove that a metric space is sequentially compact $\Leftrightarrow$ it satisfies the Bolzano- Weierstrass property.

## Section: B

**Answer any one of the following:** **(5 Marks)**

3. Define a normal space and show that every compact Hausdorff- space is normal.

4. State and prove Stone -Weierstrass theorem.