Dr. B. R. Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc (Mathematics), 2022-23

Course -6, Functional Analysis

Assignment-1

Max Marks : 15 Mim. Marks : 6

Section - A

Answer any one of the following : (10 Marks)

1 . (a) State and prove Holder's inequality for integrals.

(b) State and prove Minkowski's inequality integrals.

2. Let N, N' be normed linear spaces and T be a linear transformation of N onto N'. Then T is one-to-one and T^{-1} is bounded $\Leftrightarrow$ there is a positive real number m such that $m\|x\| \leq \|Tx\|$, *for all* $x \in N$.

Section – B

Answer any one of the following: (5 Marks)

3. Every normed linear space can be embedded in a Banach space.

4. State and prove Hahn –Banach theorem.

Dr. B. R. Ambedkar Open University

Faculty of Science , Department of Mathematics

II yr M.Sc (Mathematics), 2022-23

Course – 6, Functional Analysis

Assignment - 2

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following : **(10 Marks)**

1. State and prove Cauchy-Schwarz Inequality.

2. State and prove the theorem on Gram Schmidt orthonormalization process.

Section-B

Answer any one of the following: **(5 Marks)**

3. (i) State and prove the Pythagaros theorem for inner product spaces.
   (ii) Show that any orthogonal set of nonzero vectors in an inner product space is independent.

4. State and prove Bessel's inequality.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II year M.Sc (Mathematics) 2022-23

Course-7, Complex Analysis

Assignment-1

Max.Marks:15 Min.Marks:6

Section: A

Answer any one of the following: (10 Marks)

1. (a) Prove that (i) $f(z) = |z|$ is continuous everywhere and differentiable nowhere.
   (ii) $f(z) = |z|^2$ is differentiable at origin but nowhere else.

   (b) If f is analytic on a region R and its modulus is constant on R show that f is constant on R.

2. (a) Define an analytic function. Derive Cauchy – Riemann equations.

   (b) Show that the function $f(z) = xy + iy$ is nowhere analytic.

Section: B

Answer any one of the following: (5 Marks)

3. Describe the values of *z* for which $\left|\frac{z-3}{z+3}\right| = 2.$
4. Derive Cauchy's integral formula and evaluate $\int_c \frac{e^{iz}}{z^3} dz$ where *C* is the circle $|z| = 2$.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II year M.Sc (Mathematics) 2022-23

Course-7, Complex Analysis

Assignment-2

Max.Marks:15 Min.Marks:6

Section: A

Answer any one of the following: (10 Marks)

1. (a) Derive the Taylor series expansion of a function $f(z)$ at a point z_0.

   (b) State and prove the Residue Theorem.

2. State and prove argument principle. Deduce Rouche's theorem.

Section: B

Answer any one of the following: (5 Marks)

3. (a) Show that the function $\frac{\sin^2 z}{z^2}$ has a removable singularity at $z = 0$.

   (b) Expand $\frac{1}{z}\cosh\frac{1}{z}$ about 0 by using Laurent series and name the type of singularity.

4. Show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+4)^3} dx = \frac{\Pi}{64}$.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II year M.Sc (Mathematics) 2022-23

Course – 8A, Commutative Algebra

Assignment-1

Max.Marks :15 Min.Marks : 6

Section-A

Answer any one of the following: (10 Marks)

1. Let P be an ideal of a ring R. Suppose P is not finitely generated and is maximal among all ideals of R that are not finitely generated. Then prove that P is a prime ideal .

2. (a) Let M be an R- module and N a nonempty subset of M. Then prove that N is an R-submodule of M if and only if $ax + by \in M$ for all $a, b \in R$ and $x, y \in N$.

   (b) State and prove fundamental theorem for R – module homomorphisms.

Section- B

Answer any one of the following: (5 Marks)

3. Define the concept of a p – ring and prove that every p – ring is commutative for all primes p.

4. Define the prime spectrum of a ring R and prove that it is a compact and T_0 – space.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc(Mathematics) 2022-23

Course – 8A, Commutative Algebra

Assignment-2

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following: (10 Marks)

1. State and prove 2^{nd} Uniqueness theorem.
2. Define Noetherian module and show that an *R*-module *M* is Noetherian if and only if every submodule of *M* is finitely generated.

Section-B

Answer any one of the following: (5 Marks)

3. Every irreducible ideal of a Noetherian ring is primary.
4. Prove that every discrete valuation ring is a valuation ring.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II year M.Sc (Mathematics) 2022-23

Course-9A, Measure and Integration

Assignment-1

Max.Marks:15 Min.Marks:6

Section: A

Answer any one of the following: (10 Marks)

1. Prove that the Lebesgue measure is a countable additive set function on the class $\mathcal{M}$ of Lebesgue measurable sets.

2. State and prove Fatou's lemma and Deduce Lebesgue monotone convergence theorem.

Section: B

Answer any one of the following: (5 Marks)

3. Prove that the smallest σ - algebra containing the class of all open intervals is the class $\mathcal{B}$ of Borel sets.

4. State and prove Egorov's theorem.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II year M.Sc (Mathematics) 2022-23

Course-9A, Measure and Integration

Assignment-2

Max.Marks:15 Min.Marks:6

Section: A

Answer any one of the following: **(10 Marks)**

1. Show that if f is integrable in $[a,b]$ then $F(x) = \int_a^x f(t)\, dt$ is absolutely continuous on $[a,b]$.
2. State and prove Lebesgue's dominated convergence theorem.

Section: B

Answer any one of the following: **(5 Marks)**

3. Prove that every extended real –valued constant function on a measurable space (X, A) is measurable.
4. State and prove Hahn decomposition theorem.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc (Mathematics) 2022-23

Course -11A, Operator Theory

Assignment-1

Max. Marks: 15 Min.Marks 6

Section-A

Answer any one of the following : (10 Marks)

1. Let H be a Hilbert space and let C be a nonempty closed convex subset of H and let x be any vector in H. Then prove that there exists a unique vector $y_0 \in C$ such that $\|x - y_0\| \leq \|x - y\|$ for all $y \in C$.

2. Prove that L_p spaces are complete.

Section-B

Answer any one of the following: (5 Marks)

3. State and prove the closed graph theorem.

4. Prove that the space C [0,1] is

   (i) a normed linear space

   (ii) a Banach space.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics,

II yr M.Sc (Mathematics) 2022-23

Course -11A, Operator Theory

Assignment-2

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following: (10 Marks)

1. State and prove Gelfond-Neumark theorem.
2. State and prove that the Volterra equation of second kind has a unique solution for continuous kernel and $f(x)$.

Section-B

Answer any one of the following: (5 Marks)

3. If I is a closed two sided ideal in X then prove that the quotient space X/I is a Banach space.
4. Show that A (D) is a Banach algebra of C (D).

Dr.B.R.Ambedkar Open University

Faculty of Science,Department of Mathematics

II yr M.Sc (Applied Mathematics) 2022-23

Course – 8B, Methods of Applied Mathematics

Assignment-1

Max Mark : 15 Min.Marks : 6

Section-A

Answer any one of the following : **(10 Marks)**

1. (a) Find the geodesics on a right circular cylinder of radius *a*.

   (b) A mass *m* is suspended at the end of a very light vertical spring with spring constant *K* and is set into vertical motion. Use Lagrange's equation to find the equation of motion of the mass.

2 . (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 y + y^3) dydx$ by changing to polar coordinates

   (b) Find the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$

Section-B

Answer any one of the following: **(5 Marks)**

3. By changing the order of integration evaluate . $\int_0^a \int_0^a \frac{x}{x^2 + y^2} dydx$.

4. Find the area bounded by the cardioid $r = a(1 + \cos\theta)$.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc (Applied Mathematics) 2022-23

Course – 8B, Methods of Applied Mathematics

Assignment-2

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following : (10 Marks)

1. (a) Prove that the outer product of two tensors is a tensor whose rank is the sum of the ranks of the two tensors.

   (b) Prove that $xJ_n^{'}(x) = nJ_n(x) - xJ_{n+1}(x)$

2 . (a) State and prove the Quotient law of tensors.

   (b) Prove that the fundamental tensor g_{ij} is a covariant symmetric tensor of orde two.

Section-B

Answer any one of the following : (5 Marks)

3. Find the angle between the tangents to the curve $\vec{R} = t^2\vec{i} + 2t\vec{j} - t^3\vec{k}$ at the points $t = \pm 1$.

4. Solve the integral equation $y(x) = 1 + x + \int_0^x (x-t)y(t)dt$, $(0 \le x < +\infty)$.

Dr.B.R.Ambedkar Open University
Faculty of Science, Department of Mathematics
II yr M.Sc (Applied Mathematics) 2022-23
Course – 9B, Numerical Methods
Assignment-1

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following : (10 Marks)

1. Derive Gauss forward formula .Use this formula to find the value of u for x = 3.75, given the data

x	2.5	3.0	3.5	4.0	4.5	5.0
u	24.145	22.043	20.225	18.644	17.262	16.047

2. Find the least square approximation by a second degree polynomial for the following data.

x	-2	-1	0	1	2
$f(x)$	15	1	1	3	19

Section-B

Answer any one of the following: (5 Marks)

3. Represent $f(x) = 9x^2 + 11x + 5$ and its successive differences in factorial notation when h =1.

4. Derive Lagrange's interpolation formula.

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc (Applied Mathematics) 2022-23

Course -9B, Numerical Methods

Assignment-2

Max Marks : 15 Min.Marks : 6

Section-A

Answer any one of the following : (10 Marks)

1. Solve

$$4x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 - 2x_3 = 4$$
$$3x_1 + 2x_2 - 4x_3 = 6$$

using decomposition method.

2. Explain Runge–Kutta fourth order formula. Using it estimate $y(0.1)$ given that

$$\frac{dy}{dx} = y - x, y(0) = 2. \text{ Take } h = 0.1$$

Section-B

Answer any one of the following : (5 Marks)

3. Using Euler- Maclurin formula ,find the value of $log_e 2$ from $\int_0^1 \frac{dx}{1+x}$.

4. Find the Green's function for the differential $y'' - a^2 y = 0, (a \neq 0)$ and $y(0) = 0$ and $y(1) = 0$.

Dr.B.R.Ambedkar Open University
Faculty of Science, Department of Mathematics
II yr M.Sc (Mathematics/Applied Mathematics) 2022-23
Course – 10 A & B, Operations Research
Assignment-1

Max Marks : 15 Min. Marks : 6

Section-A

Answer any one of the following: **(10 Marks)**

1. Solve the following LPP using simplex method,

   Maximum $Z = 2x_1 + x_2$

   Subject to: $4x_1 + 3x_2 \le 12$, $4x_1 + x_2 \le 8$, $4x_1 - x_2 \le 8$ and $x_1\ x_2 \ge 0.$

2. Use duality to solve the following LPP:

   Minimize $Z = 2x_1 + 2x_2$

   Subject to constraints :

   $2x_1 + 4x_2 \ge 1,\ x_1 + 2x_2 \ge 1,\ 2x_1 + x_2 \ge 1; x_1, x_2 \ge 0.$

Section-B

Answer any one of the following : **(5 Marks)**

3. Solve graphically the following LPP:

   Maximize $Z = 3x_1 + 2x_2$

   Subject to the constraints: $-2x_1 + x_2 \le 1$, $x_1 \le 2$, $x_1 + x_2 \le 3$, $x_1, x_2 \ge 0.$

4. Solve the following system of linear equations , using The Gauss elimination method:

   $2x_1 + x_2 + 4x_3 = 16;\ 3x_1 + 2x_2 + x_3 = 10;\ x_1 + 3x_1 + 3x_3 = 16.$

Dr.B.R.Ambedkar Open University

Faculty of Science, Department of Mathematics

II yr M.Sc (Mathematics/Applied Mathematics) 2022-23

Course – 1O A & B , Operations Research

Assignment-2

Max Marks : 15 Min.Marks:6

Section-A

Answer any one of the following: **(10 Marks)**

1. Consider the problem of assigning four jobs J_1,J_2, J_3,J_4 to four machines M_1,M_2,M_3,M_4. The assignment costs are given below. Determine the optimal assignment schedule.

	J_1	J_2	J_3	J_4
M_1	15	13	14	17
M_2	11	12	15	13
M_3	13	12	10	11
M_4	15	17	14	16

2. Devide a positive quantity *b* into *n* parts so as to minimize their product.

Section-B

Answer any one of the following : **(5 Marks)**

3. Consider a situation where the mean arrival rate (λ) is one customer every 4 minutes and the mean service time (1/µ) is 2.5 minutes. Calculate the average number of customers in the system, average queue length , the average time a customer spends in the system and the average time a customer waits before being served.

4. In a distribution which is exactly normal 31% of the items are under 45 and 8 % of the item are over 64. What are the mean and the standard deviation of the distribution?