Dr. B. R. Ambedkar Open University
Faculty of Science, Department of Mathematics
II yr M.Sc (Mathematics), 2021-22

Course -6, Functional Analysis
Assignment-1

Max Marks : 15 Min.Marks: 6

Section - A

I. Answer any one of the following questions. (10 Marks)

1 . (a) State and prove Holder's inequality
(b) State and prove Minkowski's inequality.

2. Let T be a linear transformation of a normed normed linear space N into another normed linear space N'.Show that the following are equivalent :
(a) T is continuous
(b) T is continuous at the origin., that is, $x_n \to 0 \Rightarrow T(x_n) \to 0.$
(c) There exists a real number M≥0 such that $\|Tx\| \le M\|x\|$ for all x in N , that is , T is bounded.
(d) If $S = \{x : \|x\| \le 1\}$ is the closed unit sphere in N, then its image T(S) is bounded in N'.

Section - B

II. Answer any one of the following questions. (5 Marks)

3. Let C_0 denote the linear subspace which consists of all the subsequences that converge to zero. Show that $C_0^* = l_1$.

4. State and prove Hahn –Banach theorem.

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II yr M.Sc (Mathematics), 2021-22

Course – 6, Functional Analysis

Assignment - 2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. State and prove the Completion theorem for inner product spaces.
2. State and prove the theorem on Gram Schmidt orthonormalization process.

Section-B

II. Answer any one of the following: (5 Marks)

3. (i) State and prove the Pythagoras theorem for inner product spaces.
   (ii) Show tha any orthogonal set of nonzero vectors in an inner product space is independent.
4. (i) State Bessel's inequality.
   (ii) State Riesz Fischer theorem.
   (iii) State and prove the projection theorem.

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II year M.Sc (Mathematics) 2021-22

Course - 7, Complex Analysis

Assignment - 1

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10 Marks)

1. (a) Define an analytic function. Derive Cauchy – Riemann equations.

   (b) Show that the function $f(z) = e^y . e^{ix}$ is nowhere analytic.

2. Derive Cauchy's integral formula. And evaluate $\int_c \frac{z^2+3z+5}{z+1}\, dz \; when \; c : \; |z-a| = r.$

Section: B

II. Answer any one of the following: (5 Marks)

3. Find all possible values of (i) $(1+i)^{(1+i)}$ (ii) $(i)^{(i)} = 2$

4. Define entire function. Prove that a bounded entire function is constant.

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Course - 7, Complex Analysis
Assignment - 2

Max.Marks:15 **Min.Marks:6**

Section: A

I. Answer any one of the following: **(10 Marks)**

1. (a) Derive the Laurent series expansion of a function $f(z)$ at a point z_0 .

   (b) Expand $\frac{z^2-2z+5}{(z-2)(z^2+1)}$ in Laurent series valid in the region $1 < |z| < 2$.

2. State and prove argument principle. Deduce Rouche's theorem.

Section: B

II. Answer any one of the following: **(5 Marks)**

3. (a) Find the singular points and nature of the function $\frac{cos z}{z^3}$.

   (b) Find the singularities and residue at each of these singularities of the function $f(z) = \frac{1}{z^4+1}$.

4. Show that $\int_0^\infty \frac{x^2}{(x^2+a^2)^3} = \frac{\pi}{16a^3}$.

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Course – 9A, Measure and Integration

Assignment - 1

Max.Marks : 15 Min.Marks : 6

Section- A

I. Answer any one of the following: (10 Marks)

1. Prove that the Lebesgue measure is a countable additive set function on the class $\mathcal{M}$ of Lebesgue measurable sets.
2. (i) State and prove the bounded convergence theorem.

   (ii) Show that if f is Riemann integrable on $[a,b]$ then it is Lebesgue integrable on $[a,b]$ also.

Section-B

II. Answer any one of the following: (5 Marks)

3. Prove that the smallest σ - algebra containing the class of all open intervals is the class $\mathcal{B}$ of Borel sets.
4. State and prove Egorov's theorem.

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Course -9A, Measure and Integration

Assignment-2

Max.Marks :15 Min.Marks : 6

Section - A

I. Answer any one of the following: (10 Marks)

1. Show that if f is integrable in $[a,b]$ then $F(x) = \int_a^x f(t)\, dt$ is absolutely continuous on $[a,b]$.

2. (i) State and prove Fubini's theorem.

   (ii) State and prove Minkowski's inequality. For $L^p(\mu)$ spaces.

Section: B

II. Answer any one of the following: (5 Marks)

3. Prove that every extended real – valued constant function on a measurable space (X, A) is measurable.

4. State and prove Hahn decomposition theorem.

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Course – 8A, Commutative Algebra

Assignment-1

Max.Marks :15 Min.Marks : 6

Section - A

I. Answer any one of the following: (10 Marks)

1. Let P be an ideal of a ring R. Then prove that P is a prime ideal if and only if there is a multiplicatively closed set S in R such that P is maximal among the ideals which are disjoint from S.

2. (a) State and prove fundamental theorem for R – module homomorphisms.

   (b) Prove that a sub module N of a module M is a direct summand if and only if there is an idempotent endomorphism f of M such that $f(M) = N$.

Section- B

II. Answer any one of the following: (5 Marks)

3. Define the concept of a p – ring and prove that every p – ring is commutative for all primes p. .

4. Define the prime spectrum of a ring R and prove that it is a compact and T_0 – space.

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II yr M.Sc(Mathematics) 2021-22
Course – 8A, Commutative Algebra
Assignment-2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. Define Noetherian module and show that an R-module M is Noetherian if and only if every submodule of M is finitely generated.

2. State and prove the Hilbert's Basis theorem.

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Prove that every discrete valuation ring is a valuation ring.

4. State and prove the Artin-Rees theorem.

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II yr M.Sc (Mathematics) 2021-22

Course -11A, Operator Theory

Assignment-1

Max Marks: 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. Let H be a Hilbert space and let C be a nonempty closed convex subset of H and let x be any vector in H. Then prove that there exists a unique vector y_0 ЄC such that $\|x - y_0\| \leq \|x - y\|$ for all y ЄC

2. Prove that L_p spaces are complete..

Section-B

II. Answer any one of the following questions. (5 Marks)

3. State and prove the closed graph theorem.

4. Prove that the space $C\,[0,1]$ is

    (i) a normed linear space

    (ii) a Banach space.

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Course -11A, Operator Theory

Assignment-2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. State and prove Gelfond-Neumark theorem.

2. State and prove that the Volterra equation of second kind has a unique solution for continuous kernel and *f(x)*.

Section-B

II. Answer any one of the following questions. (5 Marks)

3. If *I* is a closed two sided ideal in *X* then prove that the quotient space *X/I* is a Banach space.

4. Show that *A* (*D*) is a Banach algebra of *C* (*D*).

Dr.B.R.Ambedkar Open University
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II yr M.Sc (Applied Mathematics) 2021-22
Course – 8B, Methods of Applied Mathematics
Assignment-1

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. **(10 Marks)**

1. (a) Find the geodesics on a right circular cylinder of radius *a*.

   (b) A mass *m* is suspended at the end of a very light vertical spring with spring constant *K* and is set into vertical motion. Use Lagrange's equation to find the equation of motion of the mass.

2 . (a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 y + y^3) dy dx$ by changing to polar coordinates.

   (b) Find the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$

Section-B

II. Answer any one of the following questions. **(5 Marks)**

3. By changing the order of integration evaluate $\int_0^a \int_0^a \frac{x}{x^2 + y^2} dy dx$

4. Find the area bounded by the cardioid $r = a(1 + \cos\theta)$.

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II yr M.Sc (Applied Mathematics) 2021-22

Course – 8B, Methods of Applied Mathematics

Assignment-2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. (a) Prove that the outer product of two tensors is a tensor whose rank is the sum of the ranks of the two tensors.

   (b) Prove that $xJ_n^{'}(x) = nJ_n(x) - xJ_{n+1}(x)$

2 . (a) State and prove the Quotient law of tensors.

   (b) Prove that the fundamental tensor g_{ij} is a covariant symmetric tensor of order two.

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Find the angle between the tangents to the curve $\vec{R} = t^2\vec{i} + 2t\vec{j} - t^3\vec{k}$ at the points $t = \pm 1$.

4. Solve the integral equation $y(x) = 1 + x + \int_0^x (x-t)y(t)dt \qquad (0 \le x < +\infty)$

Dr.B.R.Ambedkar Open University
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II yr M.Sc (Applied Mathematics) 2021-22
Course – 9B, Numerical Methods
Assignment-1

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. The values of x and e^{-x} are given in the following table. Find e^{-x} for x =1.7489 by using Stirling's formula.

x	1.72	1.73	1.74	1.75	1.76	1.77	1.78
e^{-x}	0.1790661	0.1772844	0.1755204	0.1737739	0.1720449	0.1703330	0.1686381

2. Fit the following data by a cubic spline and find $f(0.5)$ and $f(2.5)$. Use natural spline

x	0	1	2	3	4
$f(x)$	-8	-7	0	19	36

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Represent $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences in factorial notation when h=1.

4. Derive Newton's interpolation formula.

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Course -9B, Numerical Methods

Assignment-2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. Solve

$x_1 + x_2 - x_3 = 2$

$2x_1 + 3x_2 + 5x_3 = -3$

$3x_1 + 2x_2 - 3x_3 = 6$

using decomposition method.

3. Explain Runge–Kutta fourth order formula. Using it estimate $y\,(0.2)$ given that

$$\frac{dy}{dx} = x + y, y(0) = 1.$$

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Using Taylor's expansion ,find the solution of the I.V.P. $xy' = x - y, y(2) = 2$ at =2.1.

4. Find the Green's function for the differential $y'' - a^2 y = 0, (a \neq 0)$ and $y(0) = 0$ and $y(1) = 0$.

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Course – 10 A & B, Operations Research

Assignment-1

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. Use duality to solve the following LPP:

   Minimize $Z = 2x_1 + 2x_2$

   subject to constraints : $2x_1 + 4x_2 \geq 1, \; x_1 + 2x_2 \geq 1, \; 2x_1 + x_2 \geq 1; x_1, x_2 \geq 0.$

2. Solve the following LPP using simplex method,

   $Max \quad Z = 2x_1 + x_2 + x_3 \; subject\, to$

   $4x_1 + 6x_2 + 3x_3 \leq 8, 3x_1 - 6x_2 - 4x_3 \leq 1, \; 2x_1 + 3x_2 - 5x_3 \geq 4;$

   $x_1, x_2, x_3 \geq 0.$

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Solve the following system of linear equations , using The Gauss elimination method:

   $x_1 + 2x_2 - 2x_3 = 1; -4x_1 + 3x_2 + x_3 = -7; \; 5x_1 - x_2 + 2x_3 = 18.$

4. Solve graphically the following LPP:

   Maximize $Z = 5x_1 + 2x_2$

   subject to the constraints: $4x_1 + x_2 \geq 8, x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 2.$

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II yr M.Sc (Mathematics/Applied Mathematics) 2021-22

Course – 1O A & B , Operations Research

Assignment-2

Max Marks : 15 Min.Marks: 6

Section-A

I. Answer any one of the following questions. (10 Marks)

1. Consider the problem of assigning four jobs J_1,J_2,J3,J4 to four machines M_1,M_2,M_3,M_4. The assignment costs are given below. Determine the optimal assignment schedule:

	Machine			
Job	M_1	M_2	M_3	M_4
J_1	15	14	12	16
J_2	23	22	25	24
J_3	31	34	32	33
J_4	21	32	44	53

2. Devide a positive quantity *b* into *n* parts so as to minimize their product.

Section-B

II. Answer any one of the following questions. (5 Marks)

3. Consider a situation where the mean arrival rate (λ) is one customer every 4 minutes and the mean service time (1/μ) is 2.5 minutes. Calculate the average number of customers in the system, average queue length , the average time a customer spends in the system and the average time a customer waits before being served.

4. Use Bellman's principle to minimize $Z = y_1 + y_2 + ... + y_n$ to the constraint

$y_1 y_2 .... y_n = d, y_i > 0 \; for \;\; i = 1,2,....,n.$