Dr.B.R.Ambedkar Open University
Faculty of Science, Department of Mathematics
I year M.Sc (Mathematics) 2021-22
Course - 1, Algebra
Assignment - 1

Max.Marks:15 Min.Marks: 6

Section: A

I. Answer any one of the following: (10marks)

1. Define the concept of Left and Right Cosets. Prove that if H, K are finite subgroups of a group G, then $|HK| = \frac{|H|\,|K|}{|H \cap K|}$.
2. State and Prove the Fundamental Theorem of Homomorphism for Rings.

Section: B

II. Answer any one of the following: (5marks)

3. State and Prove Cauchy's Theorem for abelian groups.
4. Define a Ring. Give an example of a Commutative Ring with identity.

Dr.B.R.Ambedkar Open University
Faculty of Science, Department of Mathematics
I year M.Sc (Mathematics) 2021-22
Course - 1, Algebra
Assignment - 2

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10 marks)

1. Prove that every Euclidean domain is a Unique factorization domain.
2. State and prove fundamental theorem of Galois Theory.

Section: B

II. Answer any one of the following: (5 marks)

3. Prove that every field is an integral domain.
4. State and prove Eisenstein Criterion.

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Course - 2, Real Analysis

Assignment - 1

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10marks)

1. Define a complete Metric space. State and prove Cantor's Intersection Theorem.
2. If f is monotonic on $[a, b]$ and α is continuous on $[a, b]$ then $f \in R\ (\alpha\ ; a, b)$.

Section: B

II . Answer any one of the following: (5marks)

3. Define a Metric space and a Convergent sequence in a Metric space. Prove that every Convergent sequence in X is a Cauchy sequence.
4. Prove that every infinite subset E of R^K has a limit point in R^K.

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Course - 2, Real Analysis

Assignment - 2

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10marks)

1. Define Gamma and Beta functions . Prove that

$$B\,(x,y)=\frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}\ for\ all\ x>0\,,y>0.$$

2. Define Linear operator. Prove that a linear operator T on R^n is one to one if and only if T is onto. In either case T^{-1} is a linear operator.

Section: B

II. Answer any one of the following: (5marks)

3. Define Radius of the convergence of the power series. Find radius of the convergence of the power series $\sum_{n=1}^{\infty} n^n\, z^n$.

4. Let $f\,,g$ be measurable functions . Then
(i) f is integrable iff $|f|$ is integrable,
(ii) if f is integrable and g is bounded then fg is integral.

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Course - 3, Discrete Mathematical Structures

Assignment - 1

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10marks)

1. (a) Define the concept of a modular and a distributive lattices . Give one example to each.

   (b) Prove that every distributive lattice is modular. Give an example of modular lattice which is distributive.

2. (a) Define the concept of a Boolean algebra and a Boolean ring. Give one example to each.

   (b) Establish a duality between Boolean algebras and Boolean rings.

Section: B

II. Answer any one of the following: (5marks)

3. Define a Poset and give two examples. Prove that every finite poset has maximal elements as well as minimal elements.

4. (a) Define a tautology ,contradiction and continjency.

   (b) Construct the truth table of the following statements.

   (i) $(P \to Q) \wedge (\neg P \to Q)$

   (ii) $\neg(P \vee (Q \wedge R)) \leftrightarrows (P \vee Q) \wedge (P \vee R)$.

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I year M.Sc (Mathematics) 2021-22
Course - 3, Discrete Mathematical Structures
Assignment - 2

Max.Marks:15 Min.Marks:6

Section: A

I . Answer any one of the following: (10marks)

I. (a) Define Euler Graph. State and prove a necessary and sufficient condition for a graph to be an Eulerian .

(b) Discuss Chinese postman problem.

2. Let L be the language accepted by a nondeterministic finite automation . Then there exits a deterministic finite automaton that accepts L.

Section: B

II. Answer any one of the following: (5marks)

3. Define a Spanning Tree. Explain Prim's Algorithm.

4. Solve the recurrence relation $a_n = 4\, a_{n-1} + 5\, a_{n-2}$; $a_1 = 2, a_2 = 6$ by Characteristic roots method.

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Course - 4, Differential Equations

Assignment - 1

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: **(10marks)**

1. (a) Let $\Phi_1, \Phi_2, \ldots\ldots, \Phi_n$ be solutions of L(y) = 0 on I, an interval of R and $x_0 \in$ I, then

show that $W(\Phi_1, \Phi_2, \ldots\ldots, \Phi_n)(x) = W(\Phi_1, \Phi_2, \ldots\ldots, \Phi_n)(x_0)\, e^{-a_1(x-x_0)}$ $\forall\, x \in$ I.

(b) Solve the initial value problem

$$y''' - 6y'' + 11y' - 6y = 0,\ y(0) = 1,\ y'(0) = 2,\ y''(0) = 6.$$

2. For non negative integers n and m , prove that

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\, dx = \begin{cases} 0 & if\ m \neq n \\ 2^n\, n!\sqrt{\pi} & if\ m = n \end{cases}$$

Section: B

II. Answer any one of the following: **(5marks)**

3. Solve $y'' + y = \cos x$ by using annihilator method.

4. Find all periodic solutions of period 2π of the equation

$$y' + (\sin x) y = \sin x$$

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Course - 4, Differential Equations

Assignment - 2

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10marks)

1. State and prove Liouuvilli's Theorem.

2. Solve completely the equation $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial X^2}$ representing the vibration of a string of length l , fixed at both ends , given that $y(0,t) = 0,\ y(l,t) = 0,\ y(x,t) = 0,\ (\frac{dy}{dt})_{t=0} = 0,$ $0 < x < L.$

Section: B

II. Answer any one of the following: (5marks)

3. All solutions of the system $y' = A(t)\, y\,, 0 \leq t < \infty$ are stable if and only if they are bounded.

4 . (a) Formation of a partial differential equation by eliminating arbitrary constants a, b and c from $z = ax^2 + bxy + cy^2$.

(b) Solve $y^2(x-y)p + x^2(y-x)q = z(x^2 + y^2)$.

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Course - 5, Topology

Assignment - 1

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10 Marks)

1. Define Lindeloff Space . Prove that every second countable space is a Lindeloff Space.
2. State and prove Heine – Borel Theorem.

Section: B

II. Answer any one of the following: (5 Marks)

3. Suppose (X, τ) is a topological space. Let A , B be any subsets of X .*Then* show that

   (i) $\overline{\emptyset} = \emptyset$ (ii) $A \subseteq \bar{A}$ (iii) $\bar{\bar{A}} = \bar{A}$ (iv) $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

4. Prove that every convergent sequence in a metric space is a Cauchy sequence.

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Course - 5, Topology

Assignment - 2

Max.Marks:15 Min.Marks:6

Section: A

I. Answer any one of the following: (10 Marks)

1. State and prove Urysohn's Lemma.

2. Prove that
   (a) The components of a totally disconnected space are its points.
   (b) A Topological space X is locally connected if and only if each component of X is open.

Section: B

II. Answer any one of the following: (5 Marks)

3. Define Hausdorff space. Prove that every compact subspace of a Hausdorff space is closed.

4. State and prove Stone -Weierstrass theorem.