Dr. B.R. Ambedkar Open University

Faculty of Science,

DEPARTMENT OF MATHEMATICS

IInd Year M.Sc. (Mathematics) 2020-2021

Course - 6 **FUNCTIONAL ANALYSIS**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Show that any two *n*-dimensional normed linear spaces over the same scalar field are topologically isomorphic.
2. State and prove Hahn-Banach theorem.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Let for $n > 1, L = R^n$ and $L' = R^{n-1}$ be real linear spaces. Then show that the mapping $T : R^n \rightarrow R^{n-1}$ defined by $T(x_1, x_2 ..., x_n) = (x_1, x_2 .... x_{n-1})$ is a linear transformation from L in to L^1.
2. Let B and B' be two Banach spaces. Then prove that $B \times B'$ is linear space.

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IInd Year M.Sc. (Mathematics) 2020-2021

Course - 6 **FUNCTIONAL ANALYSIS**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. State and prove the Bessel's in equality.
2. State and prove the spectral theorem.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Show that $\int_{0}^{\infty} x^n x^{-x} dx = n^!$.
2. Let X be an inner product space. If for each closed linear subspace. M of X, M =M$^{\perp\perp}$, then show that X is a hilbert space.

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IInd Year M.Sc. (Mathematics) 2020-2021

Course - 7 **COMPLEX ANALYSIS**

Assignment -1

Max. Marks : 15 | Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. State and prove cauchy's integral formula.
2. State and prove Taylor's theorem.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Prove $f(z) = \frac{y}{x^2 + y^2}$ is harmonic and find a harmonic conjugate of it in its domain.
2. Test the uniform convergence of the series.

$$\sum_{n=1}^{\infty} \frac{Z^n}{n(n+1)} in |Z| \leq 1.$$

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Course - 7 **COMPLEX ANALYSIS**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Define an Entire function. State and prove weierstrass factorization theorem.

2. By integrating $\frac{(\log z)^2}{1+z^2}$ along suitable contour, prove that $\int_0^\infty \frac{(\log x)^2}{1+x^2}dx = \frac{\Pi^2}{8}$ and $\int_0^\infty \frac{\log x}{1+x^2}dx = 0$

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Evaluate $\int_0^\infty \frac{dx}{1+x^6}$.

2. For $a > b > 0$, Evaluate $\int_0^\infty \frac{\cos 2\mathrm{a}\,x - \cos 2\mathrm{b}\,x}{x^2}dx$.

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IInd Year M.Sc. (Mathematics) 2020-2021

Course - 8 A **COMMUTATIVE ALGEBRA**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Let $I_1, I_2, ..., I_n$ be ideals of ring such that

   for any $I_i + I_j = R$ for any $i \neq j$ and let . $I = \bigcap_{i=1}^{n} I_n$ Then show that

   $R/I \cong R/I_1 \; X \; R/I_2 \; X ....X \; R/I_n$

2. Prove that the maximal spectrum of any ring is compact.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Let P be an ideal of a ring R such that P is a maximal member of the class of all annihilator ideals of non-zero Elemets of R. Then show that p is prime.

2. Show that every module is a homomorphic image of a free module.

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Course - 8 A **COMMUTATIVE ALGEBRA**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Prove that an R-module P is a projective if and only if it is isomorphic to a direct summand a free R-module.
2. State and prove going - up theorem.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Show that every module is a homomorphic image of a free module.
2. Let I be an ideal of a ring R. If I has a primary decomposition, then show that I has a reduced primary decomposition.

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Course - 8 B **METHODS OF APPLIED MATHEMATICS**

<u>Assignment -1</u>

Max. Marks : 15 Min Marks : 6

<u>Section - A</u>

I Answer any one of the following Questions (10 Marks)

1. Find the stationary value of the function

$$I = \int_0^{\Pi/2} \left[2xy + \left(\frac{dx}{dt}\right)^2 \left(\frac{dy}{dt}\right)^2 \right] dt$$ satisfying

$x(0) = 0, y(0) = 0, x\left(\Pi/2\right) = -1$ and $y\left(\Pi/2\right) = -1$

2. (a) State and prove convolution theorem for Fourier transform.

   (b) Find fourier\` cosine transform of $f(x) = \dfrac{1}{1+x^2}$.

<u>Section - B</u>

II. Answer any one of the following Questions (5 Marks)

1. Solve the valterra's integral Equation.

$$y(x) = 29 + 6x + \int_o^x [5 - 6(x-t] y(t) dt.$$

2. Prove that $A_{pq} x^p x^q = 0$ If A_{pq} is a symmetric tensor.

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Course - 8 B **METHODS OF APPLIED MATHEMATICS**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. (a) If $f(x) = \exp\ (in\ x)$, Then show that

$$Z\left\{\exp\ (in\ x)\right\} = \frac{z}{z - \exp\ (ix)}$$ and deduce the

Z- transform of the sequence $\left\{\cos\ nx\right\}$

(b) Find the inverse Z- transform of $f\left(z\right)z^{n-1} = \dfrac{z(z-1)}{(z-2)^2\left(z+3\right)}$

Using the residue method.

2. Find the Zeroth order finite Hankel transform

of $a^2 - r^2$, for $0 \le r \le a$.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Prove that $\displaystyle\int_{-L}^{L} \left[f(x)\right]^2 dx = L\left[\frac{{a_0}^2}{2} + \sum_{n=1}^{\infty} \left(a_n^2 + b_n^2\right)\right]$

2. Show that the surface given by

$e^z \cos x = \cos y$ is a minimal surface.

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Course - 9 A **MEASURE & INTEGRATION**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. (a) Prove that every interval is measurable.

   (b) Deduce that every Borel set is measurable.

2. (a) Prove he existence of a bounded non-measurable set.

   (b) If E is a measurable set, prove that $m\left(E+\alpha\right)=mE$

   for every real number α

Section - B

II. Answer any one of the following Questions (5 Marks)

1. If E_1, E_2 are measurable and $m\left(E_1 U E\ \right) < \infty$.

   Then prove that ($m\left(E_1 \cup E_2\right) = m\left(E_1\right) + m\left(E_2\right) - m\left(E_1 \cap E_2\right)$.

2. If *f* is integrable on [a,b] and $F(x) = F(a) + \int_a^x f(x)\, dx$.

   then prove $F'(x) = f(x)$ *a.e.*

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Course - 9 A **MEASURE & INTEGRATION**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. State and Prove Fatou's lemma. Deduce Lebesgue monotone Convergence theorem.
2. State and prove Hahn decomposition theorem.

Section - B

II. Answer any one of the following Questions (5 Marks)

1. State and prove Caratheodry's theorem.
2. If f and g are in L^p then prove

$$\|f+g\|_p \leq \|f\|_p + \|g\|_p$$

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IInd Year M.Sc. (Mathematics) 2020-2021

Course - 9 (B) **NUMERICAL METHODS**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Derive lagrange's interpolation formula and use this formula to evaluae *u* (*x*) at *x*=6, given that

x	3	7	9	10
$u(x)$	168	120	72	63

2. Use Crout method of reduction to solve the

   System $x_1 + x_2 - x_3 = 2,\ 2x_1 + 3x_2 + 5x_3 = -3,\ 3x_1 + 2x_2 - 3x_3 = 6$

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Use Power method to find the largest eigen value and the corresponding eigen vector of the matrix.

$$\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

2. Use weddel's rule of numerical integration

   to evaluate $\int_0^6 \frac{dx}{1+x^2}$

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Course - 9 (B) **NUMERICAL METHODS**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Use modified Euler's method to find our approximate value of y at x =0.1 given that $\frac{dy}{dx} = x + y, y(0) = 1$ correct to 5 decimals - Take h=0.1 here

2. Solve $\frac{d^2y}{dx^2} + xy = -x, y\left(0\right) = y(1) = 0$ by

   Ritz - Method

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Using Crank - Nicolson Scheme, Solve the

   $PDE \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to

   $u(x,0) = 0, u(0,t) = 0$ and $u(1,\mathrm{t}) = \mathrm{t}$ choose

   $k = 1/8$ and $h = \frac{1}{2}, \frac{1}{4}$

2. Use Euler- Maclaurin formula to prove that

   $$\sum_{1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$

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Course - 10 (A&**B**) **OPERATION RESEARCH**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. Use the phase Simplex Method to solve the LPP.

   Minimize $Z = x_1 + x_2 + x_3$

   Subject to $x_1 - 3x_2 + 4x_3 = 5$

   $x_1 - 2x_2 \leq 3$

   $2x_2 - x3 \geq 4$ $x_1, x_3 \geq 0$ and x_3 is unrestricted.

2. Find an optimal sequence of the following four - Jobs five machines, when passing is not allowed, given the following processing times. Also find the total elapsed time.

Jobs				
Machine	1	2	3	4
M_1	6	5	4	7
M_2	4	5	2	2
M_3	1	3	4	2
M_4	2	4	5	1
M_5	8	9	7	5

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Solve the LPP

   Maximize $Z = x_1 + x_2$

   subject to $2x_1 + 5x_2 \leq 10$

   $4x_1 - 3x_2 \leq 6$

   $x_1 \geq 0, x_2 \geq 0$ are integers

2. Explain the steps involved in the Hungarian Assignment Algorithm

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Course - 10 (A&B) **OPERATION RESEARCH**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1) Use Kuhn - Tucker conditions to optimize

$z = 2x_1 + 3x_2 - \left(x_1^2 + x_2^2 + x_3^2\right)$ subject to constraints

$x_1 + x_2 \leq 1, 2x_1 + 3x_2 \leq 6, x_1, x_2 \geq 0$

2. Dynamic programming to solve

Minimize : $Z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Section - B

II. Answer any one of the following Questions (5 Marks)

1. Use Beale's method to solve QPP;

Maximise $Z = 2x_1 + 3x_2 - 2x_2^2$

Subject to $i) x_1 + 4x_2 \leq 4$

$ii) x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

2. Explain the basic steps in PERT/CPM techniques.

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Course - 11 (A) **OPERATOR THEORY**

Assignment -1

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

1. a) Let H be a Hilbert space and let C be a non empty closed Convex subset of H and let x be be any vector in H. Then prove that there exists a unique vector y_o in C such that $\|x - y_0\| \leq \|x - y\|$ for all y in C.

   b) Prove that it T is operator on a Hilbert space H then $(Tx, x) = 0$ for all x in H if and only if T=0.

2. a) Define $A : l^2 \to l^2$ by $A(x_1, ..., x_n .....) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, ..., \frac{x_n}{n} ... \right)$. Prove that A is Compact.

   b) Solve the integral Equation $\phi(x) = x + \frac{1}{2}\int_{-1}^{+1} (\text{t}+\text{x})\phi(x)\, dt.$

Section - B

II. Answer any one of the following Questions (5 Marks)

1) Slate and Prove Reisz representation theorem

2) State and prove Baire's Category theorem.

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Course - 11 (A) **OPERATOR THEORY**

Assignment -2

Max. Marks : 15 Min Marks : 6

Section - A

I Answer any one of the following Questions (10 Marks)

(1) (a) Let X be a Banach algebra with identity e. Let G be the set of all regular elements of X. Then prove that the mapping $\phi : G \to G$ defined by $\phi(x) = x^{-1}$ is continuous, one - one and into

(b) Let X be a Banach algebra with identity e. Let S be the set of all singular elements of X. Then prove that every boundary point of S is a topological divisor of Zero.

(2) State and prove Gelfand Representation theorem

Section - B

II. Answer any one of the following Questions (5 Marks)

1) Let X,Y be normed linear spaces. Prove that $A \in B(X, Y)$ is compact if and only $A' \in B(Y', X')$ is compact

2) Prove that the multiplication operation is continuous in a Banach algebra.